Oxford Cambridge and RSA

Wednesday 14 June 2023 – Afternoon

A Level Further Mathematics B (MEI)

Y431/01 Mechanics Minor

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space, use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 **(a)** State the dimensions of the following quantities.

- Force
- Velocity
- Density **[3]**

A student investigating the drag force F experienced by an object moving through air conjectures the formula

$$F = ku^2(\rho m^2)^{\frac{1}{3}},$$

where

- k is a dimensionless constant
- u is the air velocity relative to the moving object
- ρ is the air density
- m is the mass of the object.

(b) Show that the student's formula is dimensionally consistent. **[2]**

The student carries out experiments in an airflow tunnel. When the air density is doubled, the drag force is found to double as well, with all other conditions remaining the same.

(c) Show that the student's formula is inconsistent with the experimental observation. **[1]**

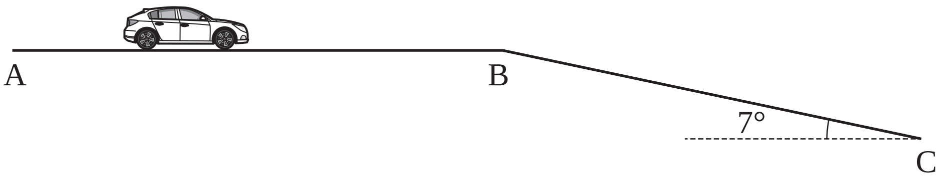
The student's teacher suggests revising the formula as

$$F = k\rho^{\alpha}u^{\beta}A^{\gamma}$$

where m has been replaced by A, the cross-sectional area of the object. The constant k is still dimensionless.

(d) Use dimensional analysis to determine the values of α, β and γ. **[3]**

2 A car of mass 1400 kg, travels along a straight horizontal road AB, after which it descends a hill BC inclined at a constant angle of 7° to the horizontal (see diagram). A, B and C all lie in the same vertical plane. Throughout the entire journey, the total resistance to the car's motion is constant.



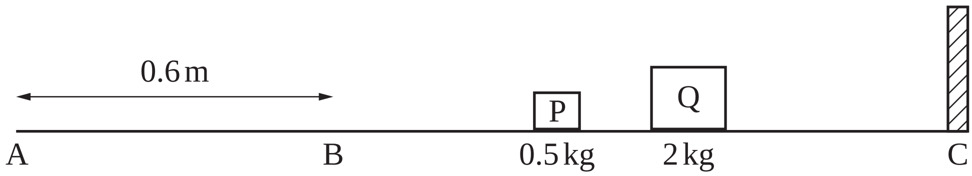
Between A and B, the car moves at a constant speed of $12\,\mathrm{m\,s^{-1}}$, and the power developed by the car is a constant *P* W. When the car reaches B, the engine is switched off and the car travels down a line of greatest slope from B to C with an acceleration of $0.8\,\mathrm{m\,s^{-2}}$. The resistance to motion is unchanged.

(a) Determine the value of *P*. **[4]**

When the car reaches C it turns round and travels back up the hill towards B at a constant speed of $v\,\mathrm{m\,s^{-1}}$. The power developed by the car between C and B is a constant 16 kW. The resistance to motion is unchanged.

(b) Determine the value of *v*. **[2]**

3 The diagram shows two blocks P and Q of masses 0.5 kg and 2 kg respectively, on a horizontal surface. The points A, B and C lie on the surface in a straight line. There is a wall at C. The surface between B and C is smooth, and the surface between A and B is rough, such that the coefficient of friction between P and AB is $\frac{2}{3}$.



P is projected with a speed of $6\,\mathrm{m\,s^{-1}}$ directly towards Q, which is at rest. As a result of the collision between P and Q, P changes direction and subsequently comes to rest at A. You may assume that P only collides with Q once.

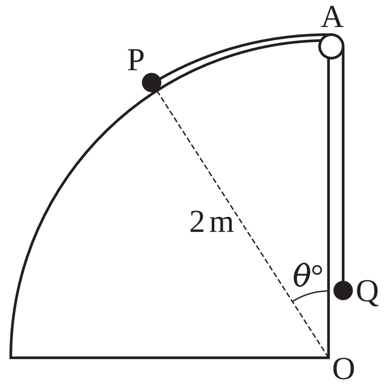
(a) Determine the coefficient of restitution between P and Q. **[6]**

(b) Calculate the impulse exerted on P by Q during their collision. **[2]**

After colliding with P, Q strikes the wall, which is perpendicular to the direction of the motion of Q, and comes to rest exactly halfway between A and B. The collision between Q and the wall is perfectly elastic.

(c) Determine the coefficient of friction between Q and AB. **[3]**

4 The diagram shows two particles P and Q, of masses 10 kg and 5 kg respectively, which are attached to the ends of a light inextensible string. The string is taut and passes over a small smooth pulley. The pulley is fixed at the highest point A on a smooth curved surface, the vertical cross-section of which is a quadrant of a circle with centre O and radius 2 m. Particle Q hangs vertically below the pulley and P is in contact with the surface, where the angle AOP is equal to $\theta°$. The pulley, P and Q all lie in the same vertical plane.



Throughout this question you may assume that there are no resistances to the motion of either P or Q and the force acting on P due to the tension in the string is tangential to the curved surface at P.

(a) Given that P is in equilibrium at the point where $\theta = \alpha$, determine the value of α. **[3]**

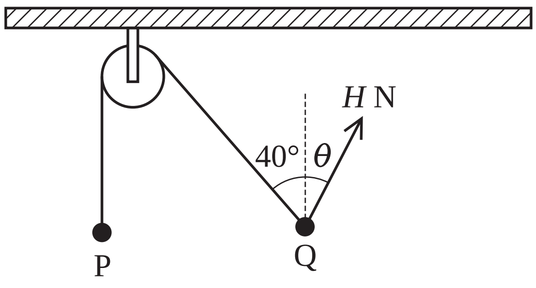
Particle P is now released from rest at the point on the surface where $\theta = 35$, and starts to move downwards on the surface. In the subsequent motion it is given that P does not leave the surface.

(b) By considering energy, determine the speed of P at the instant when $\theta = 45$. **[4]**

(c) State one modelling assumption you have made in determining the answer to part **(b)**. **[1]**

5 **Fig. 5.1** shows a particle P, of mass 5 kg, and a particle Q, of mass 11 kg, which are attached to the ends of a light, inextensible string. The string is taut and passes over a small smooth pulley fixed to the ceiling.

Fig. 5.1

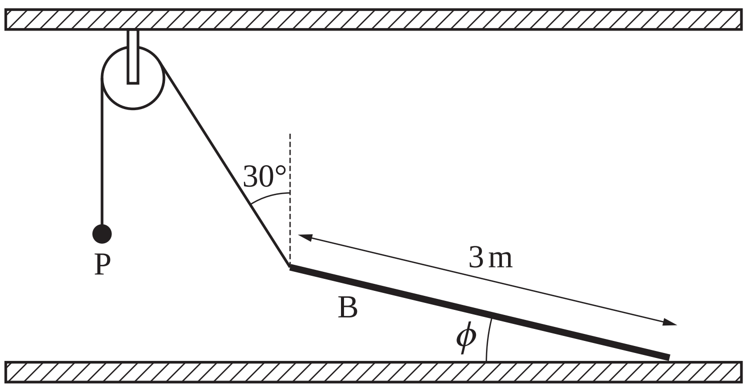


When a force of magnitude H N, acting at an angle θ to the upward vertical, is applied to Q the particles hang in equilibrium, with the part of the string connecting the pulley to Q making an angle of 40° with the upward vertical. It is given that the force acts in the same vertical plane in which the string lies.

(a) Determine the values of H and θ. **[4]**

Particle Q is now removed. The string is instead attached to one end of a uniform beam B of length 3 m and mass 7 kg. The other end of B is in contact with a rough horizontal floor. The situation is shown in **Fig. 5.2**.

Fig. 5.2



With B in equilibrium, at an angle ϕ to the horizontal, the part of the string connecting the pulley to B makes an angle of 30° with the upward vertical.

It is given that the string and B lie in the same vertical plane.

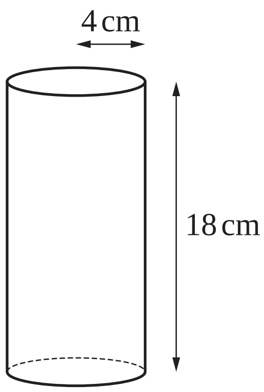
(b) Determine the smallest possible value for the coefficient of friction between B and the floor. **[3]**

(c) Determine the value of ϕ. **[5]**

6 In this question you may use the fact that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Fig. 6.1 shows a container in the shape of an open-topped cylinder. The cylinder has height 18 cm and radius 4 cm. The curved surface and the base can be modelled as uniform laminae with the same mass per unit area. The container rests on a horizontal surface.

Fig. 6.1



(a) Show that the centre of mass of the container lies 8.1 cm above its base. **[3]**

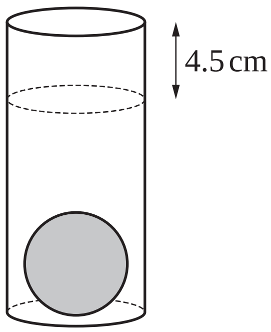
The mass of the container is 400 grams. Water is poured into the container to reach a height of h cm above the base. The centre of mass of the combined container and water lies y cm above the base. Water has a density of 1 gram per cm^3.

(b) **In this question you must show detailed reasoning.**

By formulating an expression for y in terms of h, determine the value of h for which y is lowest. **[6]**

More water is now poured into the container. A sphere of radius 3 cm is placed into the container, where it sinks to the bottom. The surface of the water is now 4.5 cm from the top of the container, as shown in **Fig. 6.2**.

Fig. 6.2



(c) Show that the centre of mass of the water in the container lies 7.5 cm above the base of the container. **[2]**

The sphere has a density of 4 grams per cm^3.

The centre of mass of the combined container, water and sphere lies z cm above the base.

(d) Determine the value of z. **[3]**

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